**Problem:**

Playing multiple games at once.

At each turn, each player chooses a game and makes a move.

You lose if there is no possible move.

**Solution:**

Grundy Numbers (Nimbers)

For each game, we compute its Grundy number

The first player wins iff the XOR of all grundy numbers is nonzero

Computing the grundy numbers:

Let S be a state, and be states reachable from S using a single move.

The Grundy number of a losing state is 0

The Grundy number g(S) of S is the smallest nonnegative integer that does not appear in

**Problem:**

Game Fifteen

Given a 4x4 grid of unique numbers between 1 and 15 and one empty cell.

The position of a number can be exchanged with the position of the empty cell if it is adjacent.

Is it possible to arrange the numbers to this permutation?

**Solution:**

Let N be the number of inversions in the permutation.

Let K be the line number (starting at zero) of the empty cell.

A solution exists iff N+K is even

**Problem:**

Build the set of all non-negative fractions.

**Solution:**

Start with the fractions:

(0/1, 1/0)

For every pair of adjacent fractions, create a new fraction between them where the numerator is the sum of their numerators and the denominator is the sum of their denominators. Repeat infinitely.

**Problem:**

Count the number of spanning trees in a graph G

**Solution:**

Kirchoff matrix theorem

Let D be the degree matrix of G

Let A be the adjaceny matrix of G

Let Q = D - A

Let Q' be the matrix resulting from deleting any row and any column from Q

The number of spanning trees is equal to the determinant of Q'